Test of Modified Newtonian Dynamics in Picogravity
The Dark Matter Alternative Solution

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Abstract

The dark matter hypothesis has been put forward to account for a discrepancy between the Newtonian dynamical mass and the observable mass in large astronomical systems. Dark matter is believed to dominate the universe and it should be in range of 90 to 99 % of its total mass. Modified Newtonian Dynamics (MOND) is an empirically motivated alternative to cosmic dark matter which satisfies an impressive proportion of observational tests. An experimental test of MOND is described, based on the mutual interaction of two bodies in an environment of very low residual gravity, such that an observable parameter of the motion, a frequency of oscillation, shall determine the possible validity of MOND.

1. Introduction.

Modified Newtonian dynamics (MOND) may provide an alternative explanation of phenomena currently attributed to dark matter. The latter is not directly detectable. However, MOND may be amenable to experimental verification via very low gravity experiments described below.

There is a discrepancy between Newtonian dynamical mass and the observable mass in large astronomical systems. One explanation is the existence of non-visible matter altering the motion of galaxies. An alternative proposed by Milgrom [1)] is MOND which has been discussed recently by Sanders and McGaugh[2)]. The effective gravitational attraction at accelerations below $a_0 \sim 10^{-10}$ m s^{-2} is assumed to be non-Newtonian and would be given by the geometric mean of g_n (the ordinary Newtonian acceleration) and a_0. For a mass M, at a distance r from its mass centre (r is beyond the mass distribution of M)

$$g_n = G\, M\, r^{-2} \qquad (1)$$

where G is the gravitational constant, the value a_o can be considered to be a natural constant, much as the velocity of light c. Table I shows a comparison of MOND with the Dark Matter hypothesis.

Table 1 MOND vs. Dark matter predictions of observables

Test	MOND	Dark Matter
Stellar mass/light ratio	yes	no
Local M/L	yes	no
Characteristic acceleration	yes	no
Transition radii	yes	no
Dwarf spheroid galaxies	yes	no
Low mass spiral galaxies[3)]	yes	no

2. Experimental test of MOND.

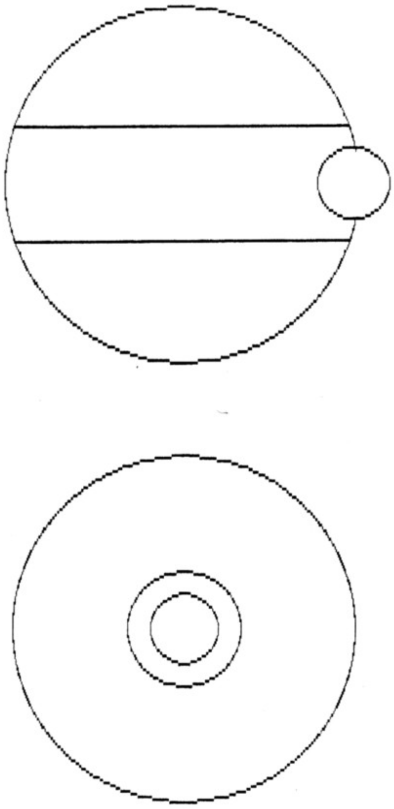
It is possible to devise a two-body system to be operated in a very low gravity environment such that the Newtonian accelerations will reach values of 10^{-11} m s^{-2} and thus should allow to test the effective accelerations mentioned above .It can be constructed of stable materials over a reasonable temperature range. For considerations related to symmetry conditions both masses of the system, a large mass **M** and a small mass **m** should be spherical. Low density materials ($\rho \sim = 50$ kg.m^{-3}) would make experimental measurements feasible in a very low gravity environment. A cylindrical tunnel is drilled along a diameter of **M** and **m** (of much smaller radius) can move freely along the tunnel as shown in **Fig. 1**.

As a first approximation the motion is harmonic because the Newtonian force within the spherical mass **M** is given .by

$$f = -\mu\, k\, r \qquad (2)$$

where $k = (4/3)\,\pi\,\rho\,G$ and ρ is the density of **M**, μ is the reduced mass of the system. When **m** << **M** μ can be safely replaced by **m** and the frequency of harmonic motion ω is simply expressed by [4)]

$$\omega^2 = k/\mathbf{m} \qquad (3)$$



.01 m

Fig. 1 Diagrams of sections of the 2-body system by planes containing the center of symmetry. Top: section along the tunnel. Bottom: section perpendicular to the previous one, with the small mass centered.

The MOND prediction has to be calculated point by point with the geometric mean of g_n with a_0.

A measurement of $\omega = 2\pi\nu$, for example via the period $\tau = 1/\nu$, would reveal a departure from Newtonian behavior and confirm (or not) the anomalous accelerations governed by the assumed value of a_0 and the geometric mean rule.

For a system of reasonable dimensions, v.l. **M** and **m** having radii of .01 m and 0.001 m respectively the motion could be studied during sufficiently long periods of time and acceleration values all the way down to zero. For the case of density $\rho = 50$ kg.m^{-3} the maximum Newtonian acceleration is 1.4 x 10^{-10} m.sec^{-2} at a radius of .01 m. It would drop below 10^{-10} m.sec^{-2} at a distance of 0.006 m from the centre. A comparison of Newtonian accelerations with those obtained using the geometric mean with a_0 for this case is shown on **Fig. 2**. There are

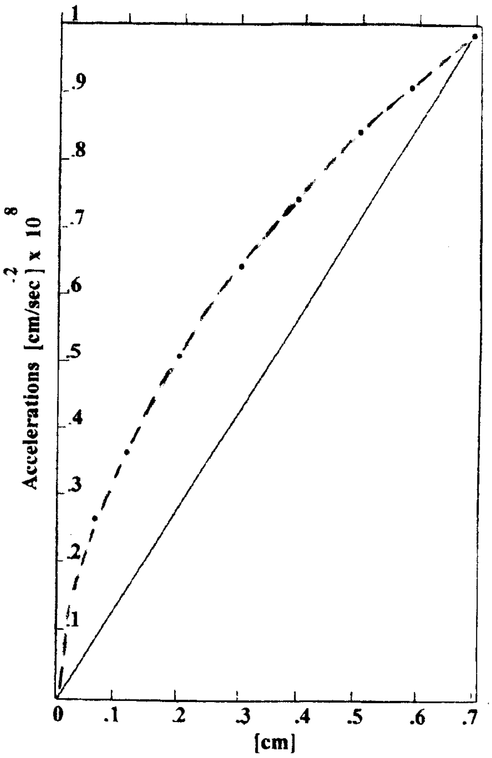
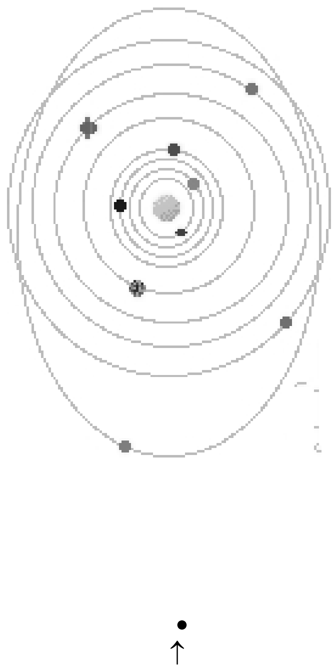


Fig. 2 The solid line corresponds to Newtonian accelerations, the dashed-dotted to the MOND prediction. At .7 cm the acceleration is Newtonian for the case discussed here.

significant differences which should result in shorter oscillation periods for the mechanical system if indeed the acceleration behaves according. to MOND below 10^{-10} m.sec^{-2} The oscillation period would be shortened by more than 40 %. The system should be contained in a ultra high vacuum box to minimize the interaction with gases. Very low values of residual vacuum could be obtained in orbit from the external environment of the Space Station, or from the outside of a space probe.

A dedicated pace probe could be launched with fully automated apparatus to be placed far from gravitational fields (**Fig. 3**) i. e. far from our planetary system. For this purpose de probe should be directed on a path perpendicular to the plane of the solar system, and stopped at a distance where gravity has dropped significantly to about 4x10^{-6} m.s^{-2}, subsequently it should drop in free fall towards the solar system. From experience on the earth based drop towers it is possible to surmise a factor of 10^{-7} with respect to the value cited above, resulting in a low gravity of the order of 10^{-12} m.s^{-2}



MOND probe

Fig. 3 Schematic of a probe far off our planetary system.

The two body system could then oscillate with small displacements of its center of mass (of the order of .01 m). Another alternative may consist in placing a satellite on an orbit based on the Euler-Lagrange points L_1 or L_2 of the Earth-Moon system (see v.l. Wiesel[5]). These are equilibrium points and the forces on the system balance each other, thus a particle at such points is stationary with no velocity or acceleration.

In all cases the experiment can be accomplished recording the motion of the system over a time equal to half a period, $\tau/2$ (about 22 hrs). Random drifts due to residual effects of gravitation should affect equally **M, m** and the dedicated space probe or satellite. Therefore the relative motion of the two body system should reflect their mutual acceleration only.

3. Discussion.

Newton's gravitation has been also questioned after a revision of the Eötvös experiments by a number of physicists[4]. The different baryonic content of elements could have altered the mutual gravitational attraction. Low gravity experiments should have provided a better environment for tests. In a certain way, MOND also questions a pure Newtonian (static) interaction in order to account for phenomena observed from large cosmic bodies (galaxies) the contention made here is that it is possible to reach the MOND type interactions with small bodies interacting within environments of extremely low gravity (nano to picogravity).

Our research group has carried out many experiments in the low gravity environment of parabolic flights and the evidence is that when g tends to zero ensembles of particles in the 1-10 μm sizes become static and no perturbations are observed over short periods of time. The bodies assumed here are clearly macroscopic but of low density and it may be permissible to expect similar behaviour.

Cosmic radiation may introduce some kind of uncertainly if it hits the interacting bodies differentially. The probe located far from the planetary system (Fig. 3) is perhaps to be preferred because, far from planetary magnetic fields, there should be no accumulation of cosmic particles, thus reducing the probability of their collisions with the probe.

In order to insure a positive outcome of the proposed measurement a number of two body systems (~10) could be prepared and observed simultaneously, providing good redundancy and allowing a statistical analysis of the results. To avoid possible radiation pressure effects the observation should be carried out with symmetrical and identical beams on either side of the two body system.

The difficulties of the experimentation sketched above should not be underestimated yet space technology has progressed significantly[6] and it should be possible to overcome them successfully.

4. Concluding remarks.

The constant a_0 postulated as a value of breakdown of the Newtonian acceleration and an alternative to dark matter, ought to be universal (scale independent) and therefore the experiments described above should put it in evidence. There is a remarkable success in the accounting of observational data with the MOND concept, paralleled by failures of the dark matter hypothesis.

Hopefully, space agencies will become interested in this fundamental research, dealing with matter in the universe. Clearly, a positive direct proof of MOND would entail a revolution in physics and a fundamental justification of such a behaviour of the gravitational force field at very low values would be necessary. Contrariwise, the dark matter hypothesis should remain an open, though strange, alternative.

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