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ガスジェット浮遊中の溶融金属液滴の表面振動特性の数値解析法

Numerical procedure for surface oscillation of aerodynamically levitated molten metal droplets

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1. Introduction

For sustainable manned activities on the moon, manufacturing with lunar regolith is essential. Especially, the melting and solidification process is necessary for the exterior walls so that they can survive in the severe lunar environment. For this purpose, preliminary numerical simulations play significant roles in the design of efficient manufacturing processes. Although the main components of the lunar regolith are metal oxides, the thermophysical properties of molten metal oxides have been measured for only a few materials. Therefore, the measurements of the thermophysical properties of molten metal oxides are an important task. For such measurements, aerodynamic levitation (ADL) is regarded as a suitable method, but there is an uncertainty problem. In ADL, the oscillatory behavior of a droplet is observed with a camera, and surface tension and viscosity are calculated based on mathematical models using the obtained frequency and damping time constant. The relationship between frequency and surface tension was analytically derived by Rayleigh1), and the relationship between the damping time constant and viscosity was derived by Lamb2). In these mathematical models, it is assumed that the droplet is a sphere, and that the system is in a stationary state on a time-averaged basis. In ADL, internal convection occurs due to the Marangoni effect from heating the droplet and the shear force from the airflow. This convection is thought to alter the frequency and damping time constant, which is believed to be the cause of the discrepancies. This study is aimed to establish the relationship between physical properties (surface tension and viscosity) and oscillation behavior (frequency and decay time constant). To achieve this goal, this study employs two approaches; 1) eigenvalue problem and 2) Volume-of-Fluid (VOF) simulation.

2. Eigenvalue problem

Even when the computational domain is restricted to two dimensions, time-dependent simulations using the Volume of Fluid (VOF) method require a substantial amount of computation for a single case, making

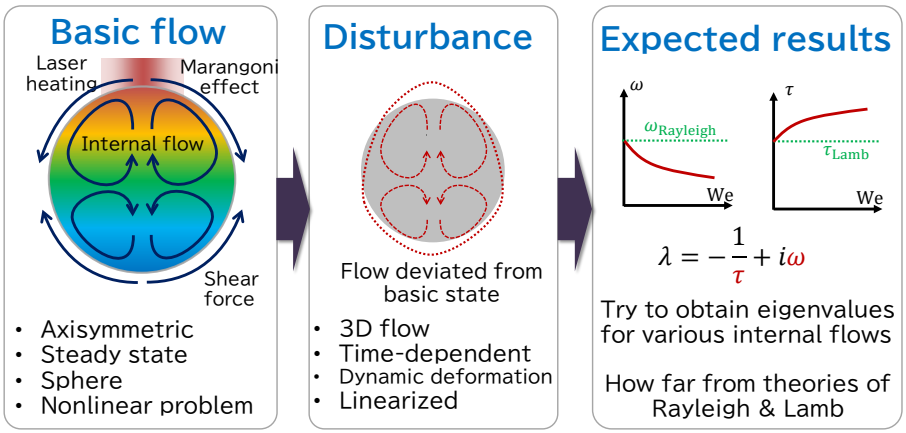

Figure 1. Problems to be considered.

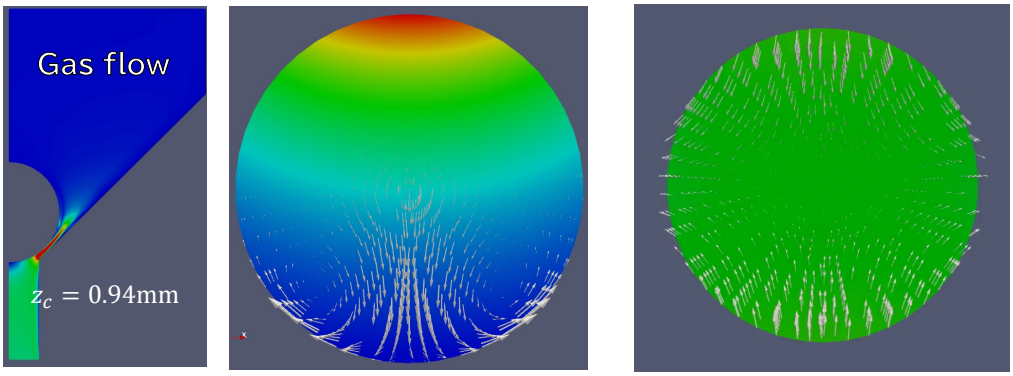

Figure 2. Surface oscillation and disturbance velocity fields.

them impractical as a model for relating oscillatory behavior to material properties. To address this limitation, we have developed a method that reformulates droplet oscillations as an eigenvalue problem, enabling the prediction of the relationship between material properties and oscillatory behavior with a computational cost dramatically lower than that of time-dependent simulations.

As illustrated in **Fig. 1**, the flow field is first decomposed into a base flow, assumed to be steady and axisymmetric, and a perturbation component. The base flow is computed as a nonlinear problem. In the basic flow, the droplet is assumed to be axisymmetric with respect to the direction of gravity. The interface position of the droplet is implicit function representation as shown in

$$F(r,\theta,\varphi,t) = r - R_0(1 + r'(\theta,\varphi,t)) \tag{1}$$

where R_0 is the radius of the corresponding sphere, and r' is the relative deformation from the perfect sphere. The flow field is divided into a basic flow and oscillatory components and is expressed in spherical coordinates as shown in Eq. (2).

$$\begin{cases} \boldsymbol{u}(r,\theta,\varphi,t) = \boldsymbol{u}_0(r,\theta) + \boldsymbol{u}'(r,\theta,\varphi,t) \\ p(r,\theta,\varphi,t) = p_0(r,\theta) + p'(r,\theta,\varphi,t) \\ \Theta(r,\theta,\varphi,t) = \Theta_0(r,\theta) + \Theta'(r,\theta,\varphi,t) \end{cases} \tag{2}$$

For spatial discretization, the spectral method is used, expanding as shown in Eq. (3).

$$u(r,\theta,\varphi) = \sum_{i}^{N_r} \sum_{l=0}^{N_\ell} \sum_{m=-\ell}^{l} u_{ilm}(t) T_i(r) Y_\ell^m(\theta,\varphi) \tag{3}$$

In the r direction, Chebyshev polynomials are used, and in the θ and φ directions, spherical harmonics are used as basis functions. For the discretized equations, the basic flow is computed iteratively using the Newton-Raphson method. Assuming the following normal modes in Eq. (4) for the linearized perturbation equations, they are solved as a generalized complex eigenvalue problem.

$$\begin{pmatrix} \boldsymbol{u}'(r,\theta,\varphi,t) \\ p'(r,\theta,\varphi,t) \\ \Theta'(r,\theta,\varphi,t) \end{pmatrix} = \begin{pmatrix} \hat{\boldsymbol{u}}(r,\theta) \\ \hat{p}(r,\theta) \\ \hat{\Theta}(r,\theta) \end{pmatrix} \exp(\lambda t + im\varphi) + \text{c.c.} \tag{4}$$

Solving the eigenvalue problem yields the decay time constant τ and the frequency ω.

To improve the fidelity of the base flow, it is necessary to explicitly compute the gas flow field around the droplet. For this purpose, we used OpenFOAM to perform gas flow simulations. The results are shown in **Fig. 2**(a), where the color contours represent the magnitude of the velocity. In actual levitation experiments, the equilibrium position of the droplet is determined by the gas flow rate and the droplet mass. In the present numerical simulations, however, the droplet position must be prescribed in advance. Therefore, simulations were conducted for different vertical positions and gas flow rates, allowing us to identify the conditions under which the vertical drag force (levitation force) balances the droplet's weight. **Fig. 3** show the oscillation frequency and temporal growth rate of the surface oscillations, respectively, for various strengths of gas-flow-induced shear. The theoretical values predicted by Rayleigh and Lamb are also indicated. The horizontal axis represents

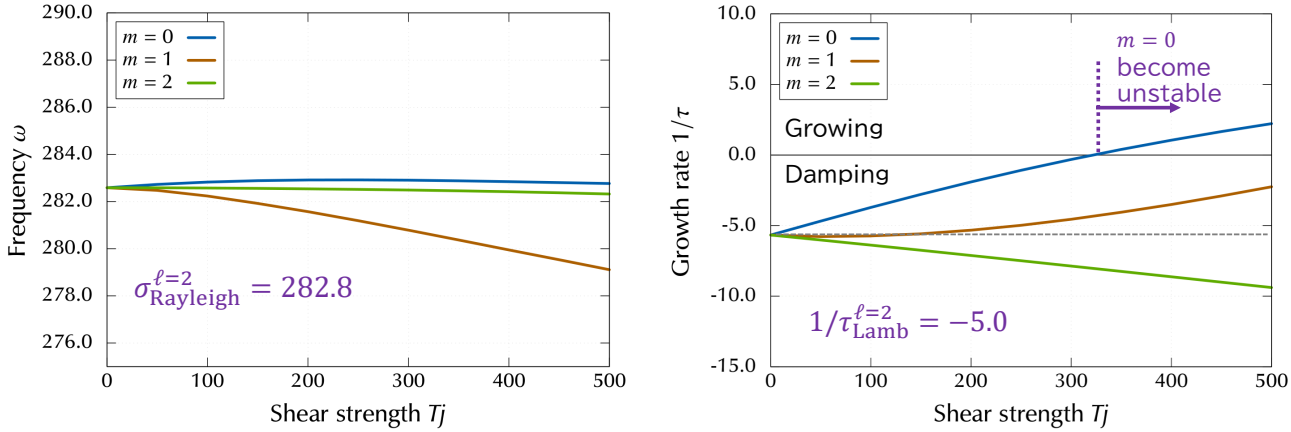

Figure 3. (left) Oscillation frequencies as a functin of shear strength. (right) Time constant for decay as a function of shear strength.

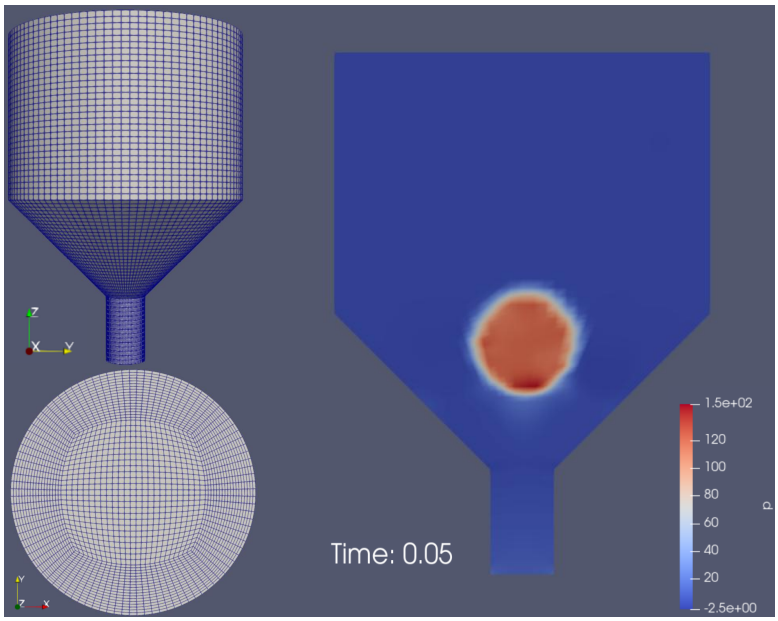

Figure 4. Computational domain and snapshot of the density field calculated by the VOF method.

the dimensionless gas shear strength, with Tj $\approx$ 300 corresponding to experimental conditions. The oscillation frequency exhibits little dependence on the gas shear strength, being only about 1% lower than the Rayleigh frequency at Tj = 300. In contrast, the growth rate varies depending on the azimuthal wavenumber m. For the $m = 2$ mode, the damping is significantly stronger than predicted by Lamb's solution, leading to an apparent viscosity about 50% higher at Tj = 300. The $m = 1$ mode shows only weak dependence on shear strength. Notably, for the axisymmetric $m = 0$ mode, the growth rate becomes positive at approximately Tj = 350, indicating that the oscillations are amplified and the base flow becomes unstable. Since the current model assumes a perfectly spherical base flow, the further development of these oscillation modes cannot be addressed at this stage. Future work should incorporate static deformation into the base flow to enable such analysis.

3. Volume-of-Fluid simulation

For the validation of the above-mentioned eigenvalue problem, time-dependent Volume-of-Fluid (VOF) simulation we also carried out. In contrast to the eigenvalue problem, where the basic state is assumes as sphere, the three-dimensional surface deformation is considered in the VOF simulation. The computational domain and results are shown in Fig. 4. Detailed results will be presented at the conference.

Acknowledgments

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