Global linear stability analysis of thermo-solutal Marangoni convection with the opposing forces under microgravity conditions

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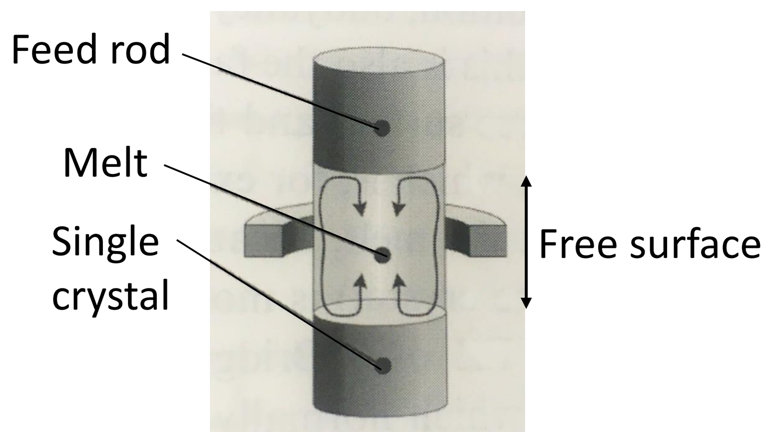
Motivation

Floating Zone (FZ) method in microgravity

- No possibility of crucible contamination
- Less gravitational segregation

Marangoni convection

- Undesirable growth striations
- Non uniformity into grown crystals



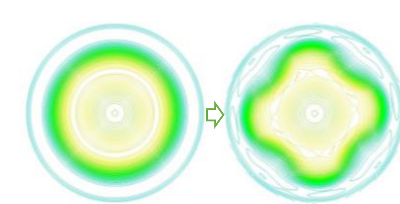
Floating Zone method – setup

(M. Lappa 2004)

Previous studies (3D CFD simulations) on Floating Zone method

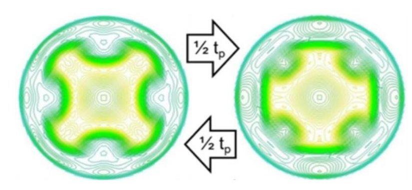
Two step flow transition

1. Steady Axisymmetric to 3D Steady flow



H. Minakuchi et al.,(2018)

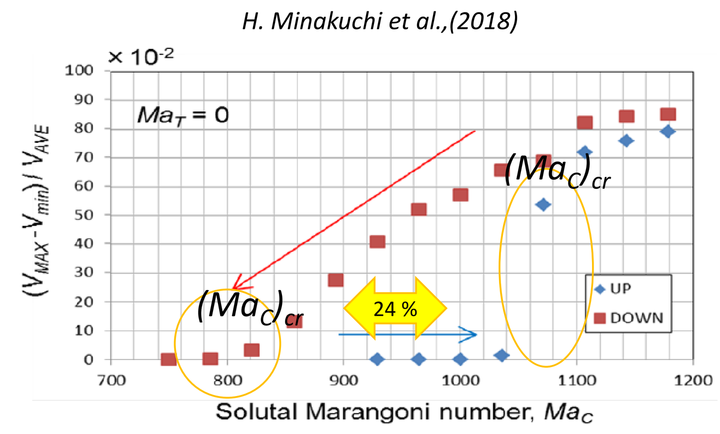
2. 3D steady to oscillating flow



H. Minakuchi et al.,(2018)

Hysteresis Study

H. Minakuchi et al.,(2018)

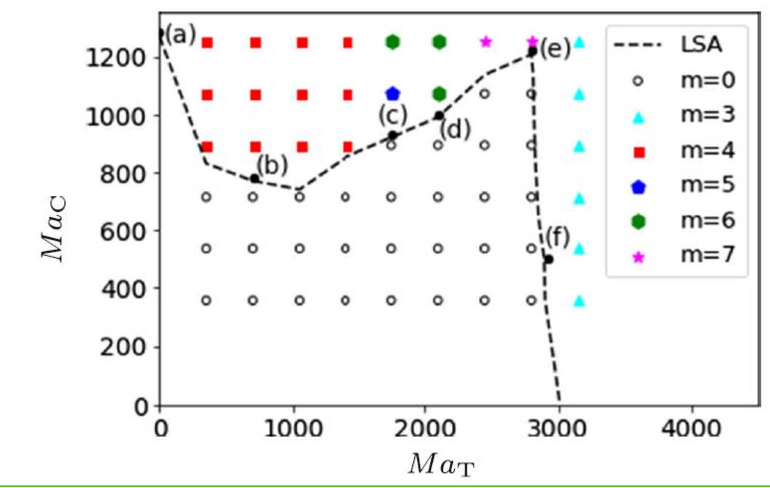


The amount of change of the velocity at sampling point $(r, \vartheta, z) = (a, 0, 0.5L)$.

- 24% difference between the critical values in the hysteresis diagram.
- The critical value $(Ma_c)_{cr}$ depends on the initial condition.

Global Linear stability analysis

R.L.A. Mendis et al.,(2020)



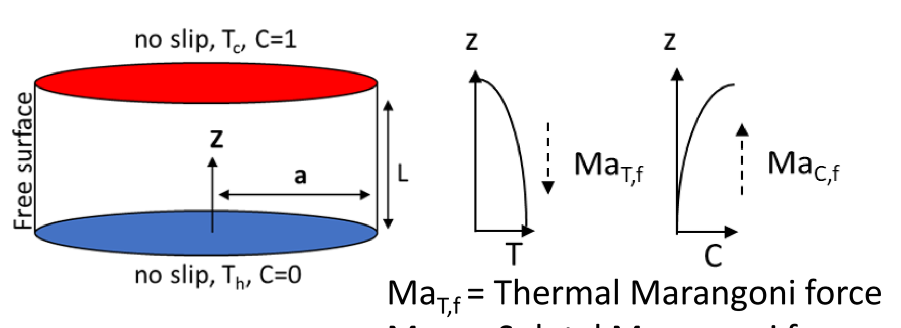
- The Onset of Thermal – Solutal Marangoni convection cannot achieve by Direct Numerical Simulations (DNS)
- The Linear stability analysis (LSA) is required to determine the Critical Marangoni numbers

Objective

The Investigation of the onset of thermal - solutal unsteady Marangoni convection with the opposing forces by linear stability analysis.

Methodology

Numerical Model



$Ma_{T,f}$ = Thermal Marangoni force
$Ma_{C,f}$ = Solutal Marangoni force

Control Parameters

$$Ma_T = \left|\frac{\partial\sigma}{\partial T}\right|\frac{\Delta T L}{\mu\nu}$$

$$Ma_C = \left|\frac{\partial\sigma}{\partial C}\right|\frac{\Delta C L}{\mu\nu}$$

Pr = 0.0067
Aspect ratio = 0.5

Governing Equations

- **Continuity**

$$\nabla\cdot\boldsymbol{u} = 0$$

- **Navier-Stokes**

$$\frac{\partial\boldsymbol{u}}{\partial t} + \mathbf{u}\cdot\nabla\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{u}$$

- **Energy**

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = \alpha\nabla^2 T$$

- **Diffusion**

$$\frac{\partial C}{\partial t} + \mathbf{u}\cdot\nabla C = D\nabla^2 C$$

where, u = velocity vector, t = time, p = pressure, T = temperature, C = Si concentration, ν = kinematic viscosity, α = thermal diffusivity, D = diffusion coefficient, ρ = density.

Global Linear Stability Analysis

Flow decomposition

$$\boldsymbol{u}(r,\theta,z,t) = \overline{\boldsymbol{u}}(r,\theta,z) + \epsilon\boldsymbol{u}'(r,\theta,z,t)$$
$$\mathrm{T}(r,\theta,z,t) = \overline{\boldsymbol{T}}(r,\theta,z) + \epsilon\boldsymbol{T}'(r,\theta,z,t)$$
$$\boldsymbol{C}(r,\theta,z,t) = \overline{\boldsymbol{C}}(r,\theta,z) + \epsilon\boldsymbol{C}'(r,\theta,z,t)$$

By Substituting In governing equations

$$\lambda\hat{\boldsymbol{u}} = A\hat{\boldsymbol{u}}$$

$\lambda > 0$ ⇒ Unstable base flow
$\lambda < 0$ ⇒ Stable base flow
$\lambda = 0$ ⇒ Onset

Arnoldi Method *F. Gómez et al., (2014)*

*Let **b** be an arbitrary initial vector*

$q_1 = b/\|b\|_2$
for $n = 1,2,3,..$
  $v = Aq_n$ ⇐ OpenFOAM $Aq_n = \dfrac{\int_0^t f(\overline{\boldsymbol{u}} + \epsilon q_n)\boldsymbol{dt} - \int_0^t f(\overline{\boldsymbol{u}})\boldsymbol{dt}}{|\epsilon|}$
  for $j = 1{:}n$
    $h_{jn} = q_j^* v$
    $v = v - h_{jn}q_j$
  end
  $h_{n+1,n} = \|v\|_2$
  $q_{n+1} = v/h_{n+1,n}$
end

Upper Hessenberg

$$\begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & h_{n+1,n} \end{bmatrix}$$

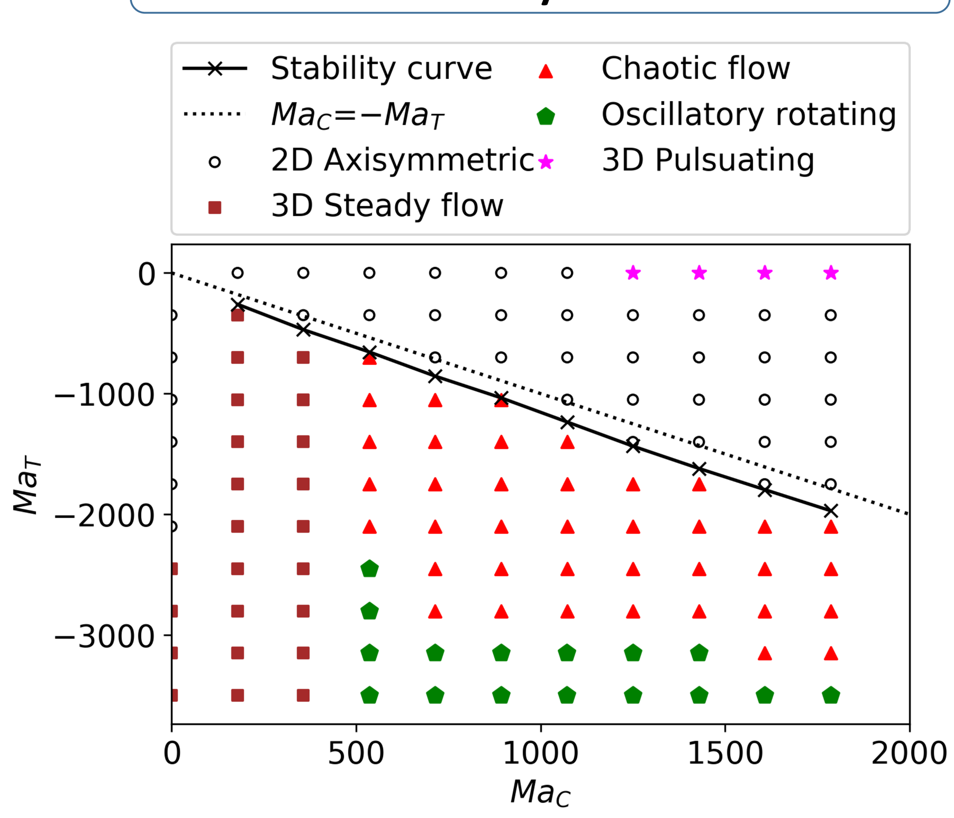
⇨ Eigenvalues of Hessenberg ≈ Leading Eigenvalues of A (λ)

Validation

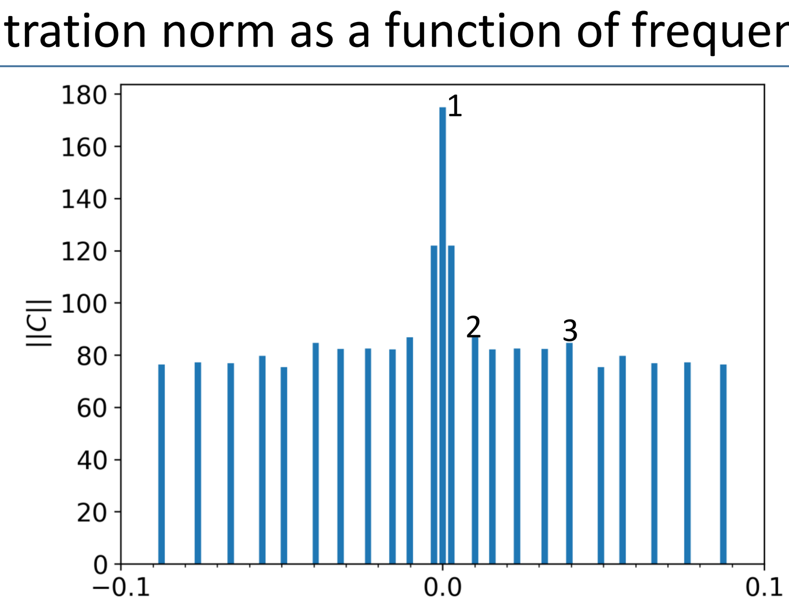
Chen, Li Ee, Roux, & Chen, 1997			Wanschura, Shevtsova, Kuhlmann, & Rath, 1995			Present Work		
Pr	$Ma_{T,cri}$	As	Pr	$Ma_{T,cri}$	As	Pr	$Ma_{T,cri}$	As
0.01	1892	1	0.01	1899	1	0.01	1887	1
0.02	2054	1	0.02	2062	1	0.02	2041	1

Results

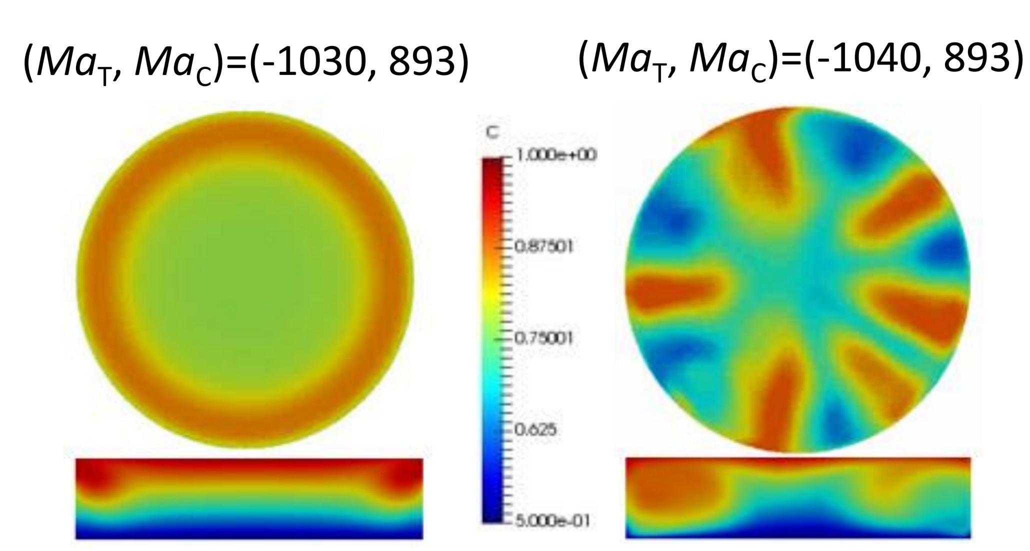
Stability Curve



Concentration norm as a function of frequency (DMD)

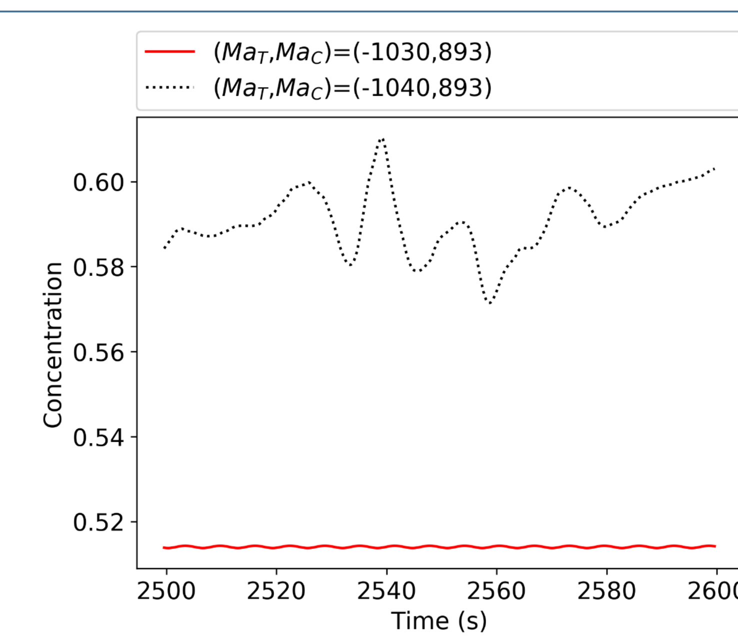


Concentration distribution of Si

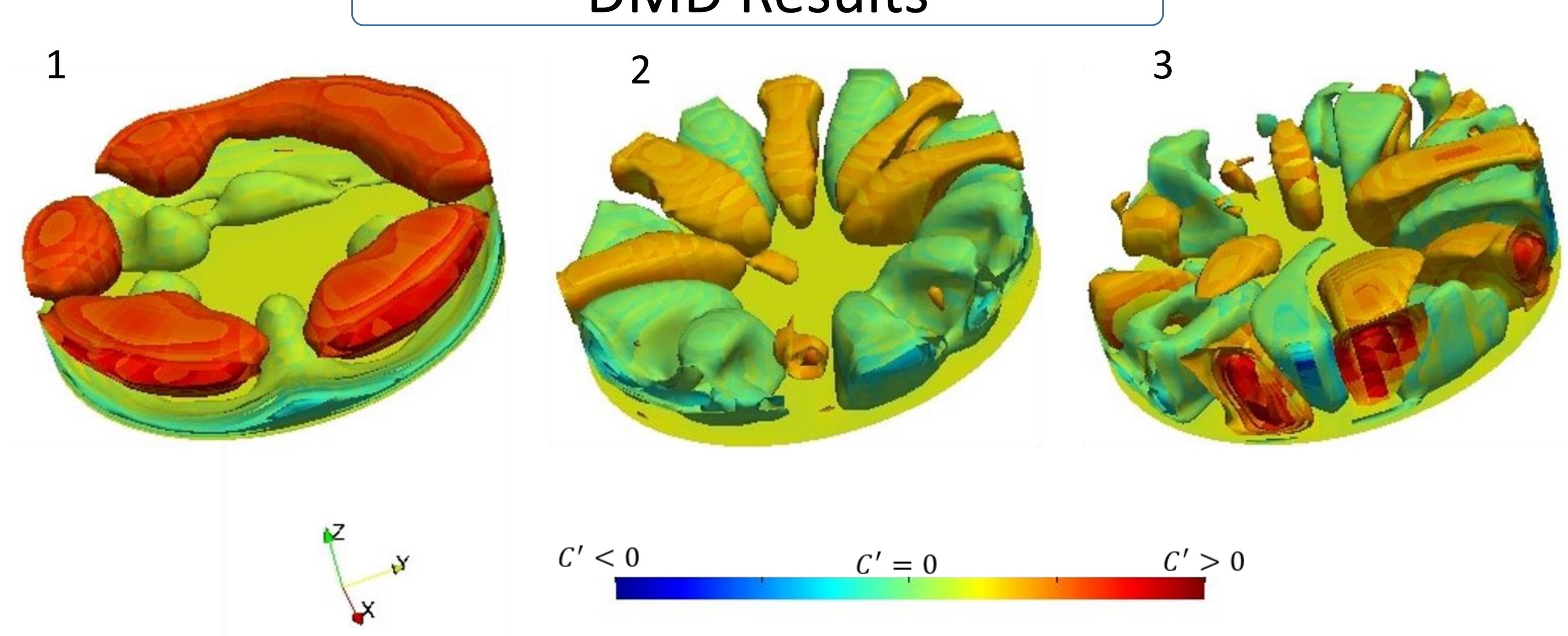


r-ϑ plane at z/L=0.5 and vertical r-z plane

Time evolution of DNS Results



DMD Results



Representative dynamic modes corresponding to the leading eigenvalues, visualized by contours of the concentration field at (Ma_T, Ma_C)=(-1040,893): (a) ω = 0 ,(b) ω = 0.01, and (c) ω = 0.039

Conclusions

- A sharp stability boundary obtained for thermal-solutal Marangoni convection.
- The stability range of the present global LSA is consistent with previous DNS results on thermal-solutal Marangoni convection with opposing forces.
- The 2D axisymmetric flow becomes chaotic through 3D steady flow when $Ma_C \le 360$. The quiescent flow directly becomes chaotic when $Ma_C > 360$.
- The Flow is 2D axisymmetric when solutal Marangoni force dominant and the flow become unstable and chaotic when $-Ma_T > Ma_C$.

Acknowledgment

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