P04

Global Linear Stability Analysis of Thermo-solutal Marangoni Convection with the Opposing Forces under Microgravity Conditions

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1. Introduction

The Marangoni flow is driven by surface tension gradients along with the liquid-gas interface due to temperature and/or concentration gradient and leads to undesirable growth striations in the grown crystals of the alloys of silicon (Si) and germanium (Ge) in the floating zone method. The previous numerical studies and linear stability analysis (LSA) [1,2] have been shown a series of transitions of flow states for low Prandtl number fluids as thermal Marangoni number increases: an axisymmetric steady flow becomes a three-dimensional non-axisymmetric flow; and then becomes an oscillatory flow. Although, the previous studies only focuse on thermal Marangoni flow, solutal one also has significant contributions to the flow pattern[3,4]. However, the theoretical onset of the thermal-solutal Marangoni convection with opposing forces has not been yet determined. Here, the present study performs three-dimensional global LSA[5] to reveal the onset of thermal and solutal Marangoni convection with the opposing forces in a cylindrical liquid bridge. The dynamic modes of the chaotic flows have been extracted as spatio-temporal coherent structures by using dynamic mode decomposition.

2. Model Formulation

A half zone model was considered as a liquid bridge between the cold and hot disks as shown in Fig. 1 assuming that (1) the fluid is incompressible and Newtonian, (2) the solid/liquid interfaces are flat and (3) the system is under zero gravity. The governing equations obtained by the mass, momentum and energy conservations are:

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u}, \tag{2}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T = \alpha \nabla^2 T, \tag{3}$$

$$\frac{\partial C}{\partial t} + \boldsymbol{u} \cdot \nabla C = D \nabla^2 C, \tag{4}$$

where, $\boldsymbol{u}$ is the velocity vector, t is the time, p is the pressure, T is the temperature, and C is the silicon concentration. ρ, $\nu(=\mu/\rho)$, α, μ, and D are the density, the kinematic viscosity, the thermal diffusivity, the viscosity, and the diffusion coefficient, respectively. Non-dimensional numbers are:

$$Ma_T = -\frac{\partial \sigma}{\partial T}\frac{\Delta T L}{\mu \nu},\ Ma_C = \frac{\partial \sigma}{\partial C}\frac{\Delta C L}{\mu \nu},\ Sc = \frac{\nu}{D},\ Pr = \frac{\nu}{\alpha},\ As = \frac{a}{L}$$

where Ma_T is the thermal Marangoni number, Ma_C is the solutal Marangoni number, Sc is Schmidt number, Pr is Prandtl number, and As is the aspect ratio of liquid bridge. In the present study Sc=14, Pr=0.006, and As=0.5 were assumed.

The boundary conditions can be expressed as follows;

along the upper disk (z=L):

non-slip condition for the velocity, T=T_C, C=1;

along the lower disk (z=0):

non-slip condition for the velocity, T=T_h, C=0;

along the free surface (r=a):

$$\boldsymbol{u}_r = 0, \mu \frac{\partial \boldsymbol{u}_z}{\partial r} = -\left(\frac{\partial \sigma}{\partial T}\frac{\partial T}{\partial z} + \frac{\partial \sigma}{\partial C}\frac{\partial C}{\partial z}\right), \ \mu\left\{r\frac{\partial}{\partial r}\left(\frac{\boldsymbol{u}_\theta}{r}\right)\right\} = -\frac{1}{r}\left(\frac{\partial \sigma}{\partial T}\frac{\partial T}{\partial \theta} + \frac{\partial \sigma}{\partial C}\frac{\partial C}{\partial \theta}\right)$$

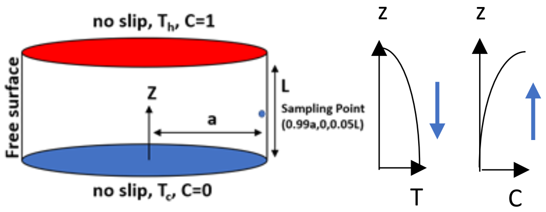

Fig 1 Geometry of liquid bridge: The arrows represent the direction of Marangoni forces by the temperature and concentration gradients along the surface

3 Linear Stability Analysis

The flow decomposes into a base flow, $\bar{\boldsymbol{u}}$, and three-dimensional infinitesimal perturbations, $(\boldsymbol{u}', C', T')$

$$\boldsymbol{u}(r,\theta,z,t) = \bar{\boldsymbol{u}}(r,\theta,z) + \boldsymbol{u}'(r,\theta,z,t) \qquad (5)$$

$$C(r,\theta,z,t) = \bar{C}(r,\theta,z) + C'(r,\theta,z,t) \qquad (6)$$

$$T(r,\theta,z,t) = \bar{T}(r,\theta,z) + T'(r,\theta,z,t) \qquad (7)$$

A linearized eigenvalue problem can be obtained by substituting this decomposition into the governing equations and by ignoring the second-order infinitesimal terms. Using a state vector $\boldsymbol{x} = (\boldsymbol{u}, T, C, p)$, we rewrite the governing equations of the variables as $\frac{\partial \boldsymbol{x}}{\partial t} = f(\boldsymbol{x})$ and the governing equation of the perturbation $\boldsymbol{x}'$ is expressed as;

$$\frac{\partial \boldsymbol{x}'}{\partial t} = \frac{\partial f(\bar{x})}{\partial x}\boldsymbol{x}' = A\boldsymbol{x}' \qquad (8)$$

where A is the Jacobian matrix which is O (10^6) in the present study. The growth rate λ with respect to the infinitesimal perturbation, $\boldsymbol{x}'(r,\theta,z,t) = \hat{\boldsymbol{x}}(r,\theta,z)\mathrm{e}^{\lambda t}$ can be obtained by solving the following eigenvalue problem;

$$\Lambda\hat{\boldsymbol{x}} = A\hat{\boldsymbol{x}} \qquad (9)$$

where Λ is the spectrum of eigenvalues determine the stability of the base flow $(\boldsymbol{x})$ and $\boldsymbol{x}$ is the corresponding eigenfunction. Here, λ>0 indicates the baseflow is unstable and λ<0 is stable. The large-scale generalized eigenvalue problem can be solved by Arnoldi method combined with time-stepping simulation using OpenFOAM [6]. The implemented linear stability code validated by comparing the first critical thermal Marangoni numbers in a literature and the results were agreed within 1% with reference values [2].

4 Results and Discussion

Figure 2 shows the stability diagram of the axisymmetric periodic base flow together with previous DNS results (on thermo-solutal Marangoni convection with the opposing forces) of Jin et al. (2020) [3]. A perodic baseflow used for LSA in The 2D axisymmetric flow becomes chaotic through 3D steady flow when solutal Marangoni number, $Ma_C \leq 360$ and quasi-periodic flow behaviour observed at the onset. The quiescent flow directly becomes chaotic when Ma_C >360. The Stability curve is laying below the line of Ma_C =- Ma_T. and thus, literally that the flow is 2D axisymmetric when solutal Marangoni force dominant. The flow becomes unstable and chaotic when – Ma_T > Ma_C. Figure 3 shows a detailed example of time snapshots of concentration distribution in horizontal and vertical planes of the liquid bridge near the onset when Ma_C=893. The time evaluation of the concentration at the sampling point ((r,θ,z)=(0.99a,0,0.5z)) shown in Fig. 4 indicates that the flow is weakly periodic and stable before it turns chaotic. The representative dynamic modes corresponding to the dominant eigenvalues with respect to the chaotic flow at (Ma_T, Ma_C) = (-1040, 893) are shown in Fig. 5. Where Fig.5 (a) represents the steady-state (mean flow) at ω = 0 and two specific modes dominate the energy spectrum are shown in Fig.5 (b) and (c) at ω = 0.01 and 0.039 rad/s.

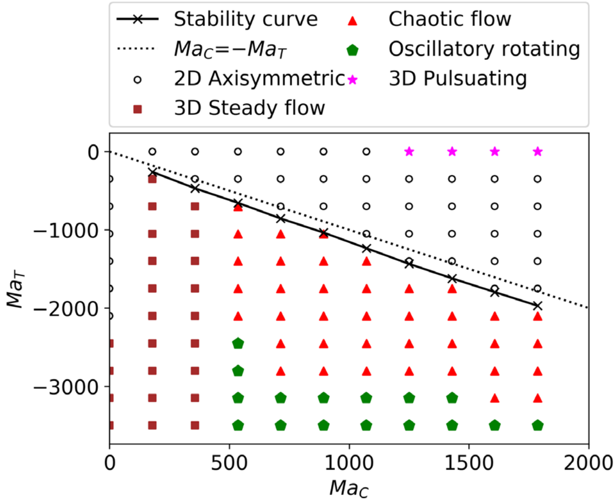

Fig 2 Stability curve of axisymmetric flow

(a) (b)

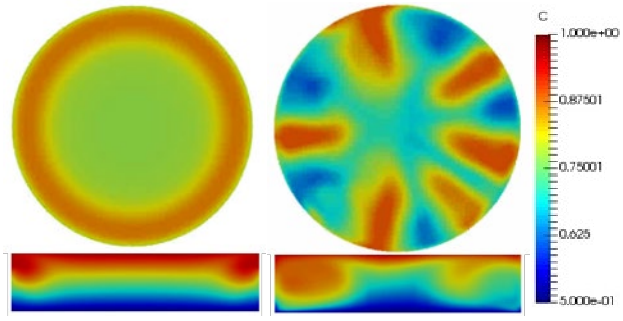

Fig 3 Concentration distribution of Si in the *r*-*θ* plane at z/L=0.5 and vertical *r*-*z* plane: (a) (Ma_T, Ma_C)=(-1030, 893) and (b) (Ma_T, Ma_C)=(-1040, 893)

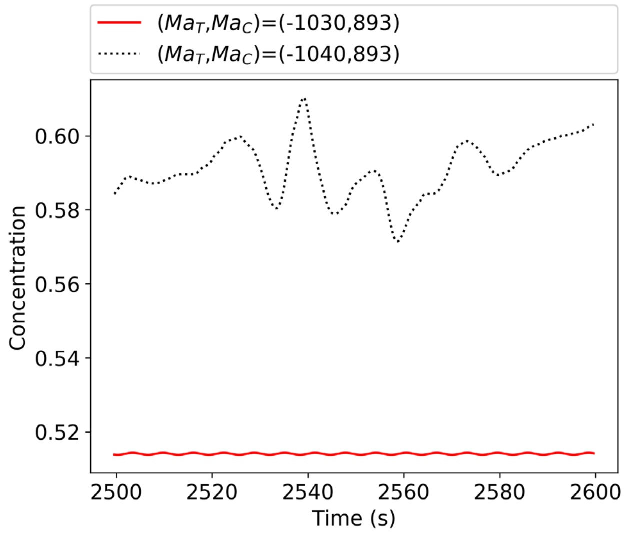

Fig 4 Time evolution of the concentration at the sampling point (r,θ,z)=(0.99a,0,0.5z)

(a) (b) (c)

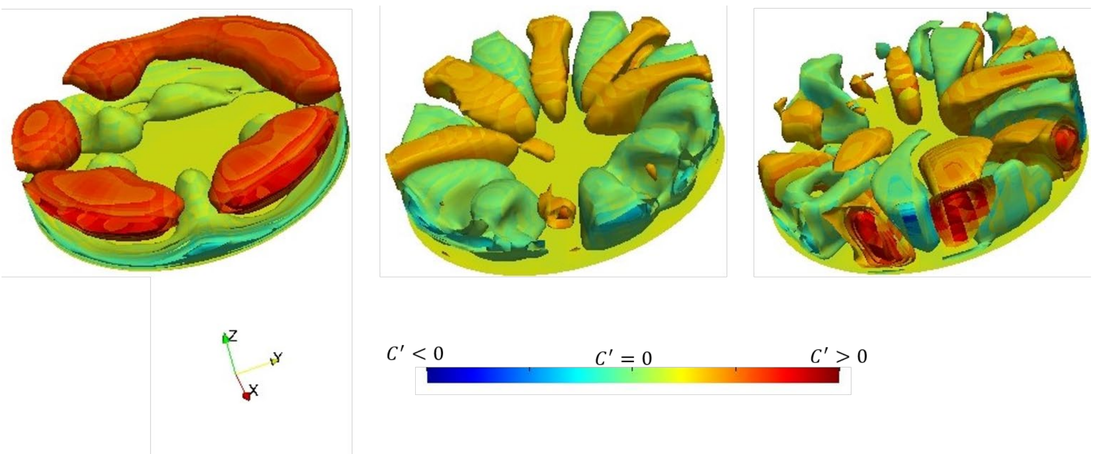

Fig 5 Representative dynamic modes corresponding to the leading eigenvalues, visualized by contours of the concentration field at (Ma_T, Ma_C)=(-1040,893): (a) ω = 0 ,(b) ω = 0.01, and (c) ω = 0.039

4 Conclusions

The onset of opposite thermo-solutal Marangoni convection in a half-zone liquid bridge is investigated by LSA. The solutal Marangoni convection develops in the direction opposite to the thermal Marangoni convection along the free surface of the liquid bridge. The 2D axisymmetric flow becomes chaotic through 3D steady flow when $Ma_C \leq 360$. The quiescent flow directly becomes chaotic when $Ma_C > 360$. The stability curve has been obtained by LSA and the representative dynamic modes corresponding to dominant eigenmodes of the chaotic flows have been extracted as spatio-temporal coherent structures by using dynamic mode decomposition.

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