Numerical simulation of thermal-solutal Marangoni convection in a shallow rectangular cavity with mutually perpendicular temperature and concentration gradients

Jiangao ZHANG, Atsushi SEKIMOTO, Yasunori OKANO

Department of Materials Engineering Science, Osaka University, Japan

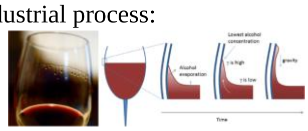
Introduction

❖ **Marangoni convection** usually takes place in a liquid layer with the free surface due to the variation of **surface tension**.

- ✓ Temperature gradient:
  → Thermal Marangoni convection
- ✓ Concentration gradient:
  → Solutal Marangoni convection
- ✓ Temperature & concentration gradients:
  → Thermal-solutal Marangoni convection

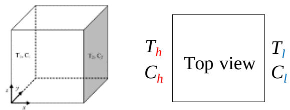
❖ **Marangoni convection** exists in various nature and industrial process:

- ✓ Tear of wine
- ✓ droplets
- ✓ Coating and Painting
- ✓ Solidification of alloy
- ✓ Crystal growth



Tear of wine[1]

[1] R Seemann, *et al*. *Eur. Phys. J. Special Topics* **225** (2016) 2227.
[2] J.M. Zhan, *et al*. *Phys. Rev. E*. **82** (2010) 066305.

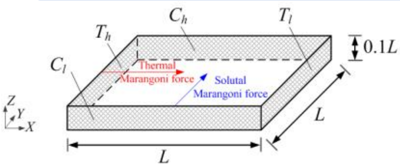


Previous physical model[2]

Mutually parallel T, C gradients → Mutually perpendicular T, C gradients

Objective: 3D numerical simulation are conducted to shed further light on the **flow characteristics** and flow **pattern transition**.

Numerical method



Physical model

Nondimensional governing Equation

Continuity: $\nabla\cdot\mathbf{V}=0$

Momentum: $\frac{\partial\mathbf{V}}{\partial\tau}+\mathbf{V}\cdot\nabla\mathbf{V}=-\nabla P+\nabla^2\mathbf{V}$

Energy: $\frac{\partial\Theta}{\partial\tau}+\mathbf{V}\cdot\nabla\Theta=\frac{1}{\Pr}\nabla^2\Theta$

Mass transfer: $\frac{\partial\Phi}{\partial\tau}+\mathbf{V}\cdot\nabla\Phi=\frac{1}{Sc}\nabla^2\Phi$

Boundary condition

$\frac{\partial V_x}{\partial z}=-Ma_T\frac{\partial\Theta}{\partial x}-Ma_C\frac{\partial\Phi}{\partial x}$ $\quad V_z=0$

$\frac{\partial V_y}{\partial z}=-Ma_T\frac{\partial\Theta}{\partial y}-Ma_C\frac{\partial\Phi}{\partial y}$

$\partial\Theta/\partial y=0$ (y=0, 1; z=0)

$\partial\Phi/\partial x=0$ (x=0, 1; z=0)

Parameter

$Asp = 0.1$

$Pr = \alpha/\nu = 0.01$

$Sc = D/\nu = 1$

$Ma_T=-(\frac{\partial\sigma}{\partial T})\frac{\Delta TL}{\mu\nu}$

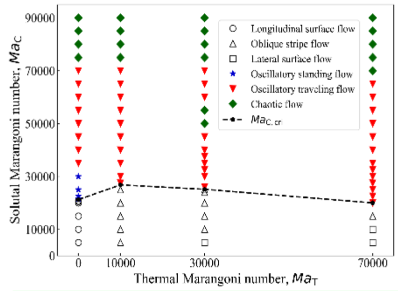
$Ma_C=(\frac{\partial\sigma}{\partial C})\frac{\Delta CL}{\mu\nu}$

$\Theta = (T- T_l)/(T_h - T_l)$

$\Phi = (C- C_l)/(C_h - C_l)$

Results and discussion

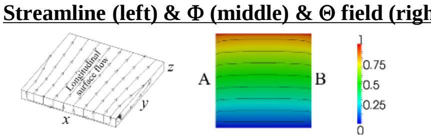
Flow map (Ma_T,Ma_C)



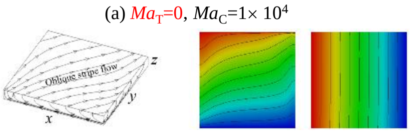
Ma_T: 0, 1, 3, 7 × 10^4 & Ma_C: 0 ~ 9 × 10^4.

Basic flow

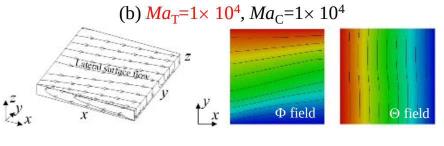
Streamline (left) & Φ (middle) & Θ field (right)



(a) Ma_T=0, Ma_C=1× 10^4



(b) Ma_T=1× 10^4, Ma_C=1× 10^4



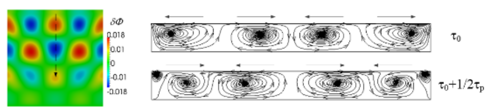
(c) Ma_T=7× 10^4, Ma_C=1× 10^4

- ➢ As Ma_T increases, basic flow evolves into three types of steady flows.
- ➢ The iso-concentration lines are distorted, while isotherms are uniform due to small Pr (= 0.01).

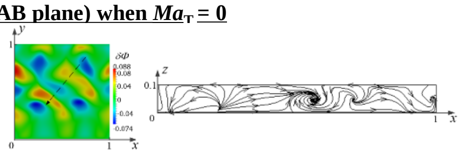
Oscillatory flow

A fluctuation quantity (δW) over one period is introduced as: (W can be Θ or Φ) → $\delta W(x,y,z)=W(x,y,z)-\frac{1}{\tau_p}\int_{\tau_0}^{\tau_0+\tau_p}W(x,y,z)d\tau$

Φ fluctuations & Streamline (AB plane) when Ma_T = 0

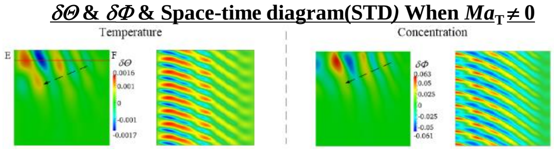


(a) Ma_T=0, Ma_C=2.15× 10^4

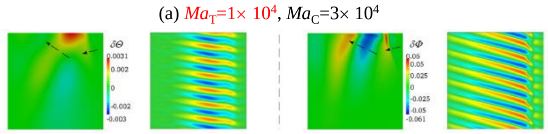


(b) Ma_T=0, Ma_C=5× 10^4

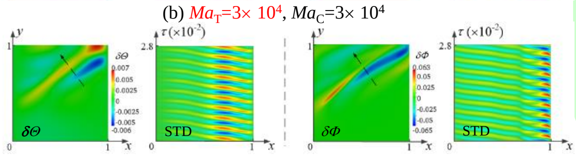
***δΘ* & *δΦ* & Space-time diagram(STD) When $Ma_T \neq 0$**



(a) Ma_T=1× 10^4, Ma_C=3× 10^4



(b) Ma_T=3× 10^4, Ma_C=3× 10^4



(c) Ma_T=7× 10^4, Ma_C=3× 10^4

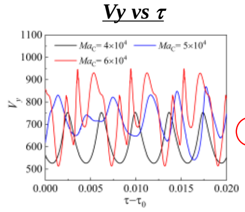
When Ma_T = 0

- ➢ Samll Ma_C → Standing wave: the periodical evolution of rolling cells (symmetry).
- ➢ Higher Ma_C → Travelling wave: a secondary instability happens (Symmetry is broken).

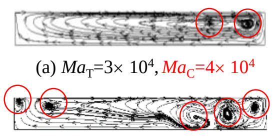
When $Ma_T \neq 0$

- ➢ $\delta\Phi$ on the free surface is similar to $\delta\Theta$
- ➢ The propagation directions of wave patterns depend on the relative contributions of thermal and solutal Marangoni effects.

Vy vs τ



Streamline(AB plane)



(a) Ma_T=3× 10^4, Ma_C=4× 10^4

(b) Ma_T=3× 10^4, Ma_C=6× 10^4

The reduction of temporal complexity (chaotic → oscillation)

The increase of spatial complexity (2 vortex to 5 vortex)

The overall complexity increases

dynamic theory

❖ Backward transition: Oscillatory → Chaotic → Oscillatory

The appearance of the backward transition from chaotic to oscillatory is reasonable.

Conclusions

(1) Flow pattern generally changes from steady to oscillatory and then to chaotic with the enhance of thermal and solutal Marangoni effect.

(2) The wave patterns of temperature and concentration fluctuations are highly depended on the coupling of thermal and solutal Marangoni effects.

(3) A backward transition phenomenon from chaotic to oscillatory are also observed.